VLADIMIR I. ARNOLD

# Collected Works

## VOLUME II

Hydrodynamics, Bifurcation Theory,
and Algebraic Geometry
1965–1972

Springer

**VLADIMIR I. ARNOLD**

# Collected Works

Vladimir I. Arnold, drawing, 1968.
*Photo by Dmitry Arnold*

# VLADIMIR I. ARNOLD

Collected Works

**VLADIMIR I. ARNOLD**

# Collected Works

VOLUME II

Hydrodynamics, Bifurcation Theory,
and Algebraic Geometry 1965-1972

# VLADIMIR I. ARNOLD

## Collected Works

VOLUME II

Hydrodynamics, Bifurcation Theory,
and Algebraic Geometry 1965-1972

Edited by

Alexander B. Givental
Boris A. Khesin
Alexander N. Varchenko
Victor A. Vassiliev
Oleg Ya. Viro

Springer

Vladimir I. Arnold

June 12, 1937 – June 3, 2010

*Editors*

Alexander B. Givental
Department of Mathematics
University of California
Berkeley, CA, USA

Victor A. Vassiliev
Steklov Mathematical Institute
Russian Academy of Sciences
Moscow, Russia

Boris A. Khesin
Department of Mathematics
University of Toronto
Toronto, ON, Canada

Oleg Ya. Viro
Institute for Mathematical Sciences
Stony Brook University
Stony Brook, NY, USA

Alexander N. Varchenko
Department of Mathematics
University of North Carolina
Chapel Hill, NC, USA

ISBN 978-3-642-31030-0 ISBN 978-3-642-31031-7 (ebook)
DOI 10.1007/978-3-642-31031-7

Library of Congress Control Number: 2013937321

© Springer-Verlag Berlin Heidelberg 2014
This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.
The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

*Cover design:* WMXDesign GmbH, Heidelberg

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

# Preface

This volume of the Collected Works appears in print after Vladimir Arnold's untimely death in June 2010. His passing was a terrible loss for mathematics and science in general.

We hope that this project of Collected Works, which is needed now more than ever, will contribute to establishing the tremendous legacy of V.I. Arnold, a remarkable mathematician and human being. Some memories of V.I. Arnold can be found in the recent March and April 2012 issues of the Notices of the AMS.

Our Editorial team has also suffered an unprecedented blow since Volume I was published in 2009. Jerry Marsden passed away in September 2010, and Vladimir Zakalyukin passed away in December 2011. We dedicate this volume to their memory.

This Volume II of the Collected Works includes papers written by V.I. Arnold mostly during the period from 1965 to 1972. This was an amazingly productive period, starting with a year Arnold spent in Paris. During this period he made fundamental contributions to the fields of hydrodynamics, algebraic geometry, singularity and bifurcation theories, and dynamical systems. We have also added later papers by Arnold on related topics so as to make this volume more complete and comprehensive.

Several papers were translated from Russian specifically for this volume. Unfortunately it was not possible to replace certain previously existing translations, completed in the era of the Iron Curtain, when it could not be expected that translators understood the subject. As an alarming example, we refer the reader to the paper “Local problems of analysis”, and to the editors' comment therein. As a counter-example, see the paper “On the arrangements of ovals...”, which has been translated once again.

For the same reason some of the titles in the reprinted translations were incorrect. Since it was not possible to correct them in the reprinted articles, we have given their correct versions in the Contents along with the versions used in the reprinted contributions.

November 2012

Alexander Givental
Boris Khesin
Alexander Varchenko
Victor Vassiliev
Oleg Viro

# Acknowledgements

The Editors thank the Göttingen State and University Library for providing the original articles for this edition, as well as D. Auroux, A. Chenciner, G. Gould and R. Montgomery for the translation and editing of several papers in this volume. They also thank the Springer office in Heidelberg, and in particular Ruth Allewelt and Martin Peters, for their extensive help and tireless support with this project.

# Contents


**1 A variational principle for three-dimensional steady flows of an ideal fluid**
*Published as "Variational Principle for three-dimensional steady-state flows of an ideal fluid" in J. Appl. Math. Mech. 29:5, 1002-1008, 1965. Translation of Prikl. Mat. Mekh. 29:5, 846-851, 1965* ..... 1

**2 On the Riemann curvature of diffeomorphism groups**
*Translation of C.R. Acad. Sc. Paris 260, 5668-5671, 1965. Translated by Denis Auroux* ..... 9

**3 Sur la topologie des écoulements stationnaires des fluides parfaits (French)**
*C. R. Acad. Sc. Paris 261, 17-20, 1965* ..... 15

**4 Conditions for non-linear stability of stationary plane curvilinear flows of an ideal fluid**
*Sov. Math. Dokl. 162, No. 5, 773-777, 1965. Translation of Dokl. Akad. Nauk SSSR, 162:5, 975-978, 1965* ..... 19

**5 On the topology of three-dimensional steady flows of an ideal fluid**
*J. Appl. Math. Mech. 30:1, 223-226 1966. Translation of Prikl. Mat. Mekh. 30:1, 183-185, 1966* ..... 25

**6 On an a priori estimate in the theory of hydrodynamical stability**
*Am. Math. Soc. Transl. (2) 79, 267-269, 1969. Translation of Izv. Vyssh. Uchebn. Zaved. Mat. 5:54, 3-5, 1966* ..... 29

**7 On the differential geometry of infinite-dimensional Lie groups and its applications to the hydrodynamics of perfect fluids**
*Translation of Annales de L'Institut Fourier, Vol. 16, No. 1, 319-361, 1966. Translated by Alain Chenciner* ..... 33

**8 On a variational principle for the steady flows of perfect fluids and its applications to problems of non-linear stability**
*Translation of Journal de Mécanique, Vol. 5, No. 1, 29-43, 1966. Translated by Alain Chenciner* ..... 71

**9 On a characteristic class arising in quantization conditions**
*Published as "Characteristic class entering in quantization conditions" in Funct. Anal. Appl. 1, 1-13, 1967. Translation of Funkts. Anal. Prilozh. 1:1, 1-14, 1967* ..... 85

**10 A note on the Weierstrass preparation theorem**
*Published as "A note on Weierstrass' auxiliary theorem" in Funct. Anal. Appl. 1, 173-179, 1967. Translation of Funkts. Anal. Prilozh. 1:3, 1-8, 1967* ..... 99

**11 The stability problem and ergodic properties for classical dynamical systems**
*Am. Math. Soc. Transl. (2) 70, 5-11, 1969. Translation of Proc. Internat. Congr. Math., Moscow 1966, 387-392. 1968* ..... 107

**12 A remark on the ramification of hyperelliptic integrals as functions of parameters**
*Published as "Remark on the branching of hyperelliptic integrals as functions of the parameters" in Funct. Anal. Appl. 2, 187-189, 1968. Translation of Funkts. Anal. Prilozh. 2:3, 1-3, 1968. Translated by Victor Vassiliev* ..... 115

**13 Singularities of smooth mappings**
*Russ. Math. Surv. 23, 1-43, 1968. Translation of Usp. Mat. Nauk 23:1, 3-44, 1968* ..... 119

**14 Remarks on singularities of finite codimension in complex dynamical systems**
*Funct. Anal. Appl. 3, 1-5, 1969. Translation of Funkts. Anal. Prilozh. 3:1, 1-6, 1969. Translated by Victor Vassiliev* ..... 163

**15 Braids of algebraic functions and the cohomology of swallowtails**
*Translation of Usp. Mat. Nauk 23:4, 247-248, 1968. Translated by Gerald Gould* ..... 171

**16 Hamiltonian nature of the Euler equations in the dynamics of a rigid body and of an ideal fluid**
*Translation of Usp. Mat. Nauk 24:3, 225-226, 1969. Translated by Gerald Gould* ..... 175

**17 On the one-dimensional cohomology of the Lie algebra of divergence-free vector fields and rotation numbers of dynamical systems**
*Published as "One-dimensional cohomologies of the Lie algebras of nondivergent vector fields and rotation numbers of dynamic systems" in Funct. Anal. Appl. 3, 319-321, 1969. Translation of Funkts. Anal. Prilozh. 3:4, 77-78, 1969. Translated by Victor Vassiliev* ..... 179

**18 The cohomology ring of the colored braid group**
*Math. Notes 5, 138-140, 1969. Translation of Mat. Zametki 5:2, 227-231, 1969. Translated by Victor Vassiliev* ..... 183

**19 On cohomology classes of algebraic functions invariant under Tschirnhausen transformations**
*Funct. Anal. Appl. 4, 74-75, 1970. Translation of Funkts. Anal. Prilozh. 4:1, 84-85, 1970. Translated by Victor Vassiliev* ..... 187

**20 Trivial problems**
*Translation of Proc. 5th Int. Conf. on Nonlinear Oscillations, Kiev 1969. Vol. 1, 630-631, Ukrain. Acad. Sciences, Kiev 1970. Translated by Gerald Gould* ..... 191

**21 Local problems of analysis**
*Moscow Univ. Math. Bull. 25 (1970), 77-80, 1970. Translation of Vestn. Mosk. Univ. Ser. I, Mat. Mekh. 25:2, 52-56, 1970* ..... 193

**22 Algebraic unsolvability of the problem of stability and the problem of topological classification of singular points of analytic systems of differential equations (Russian)**
*Usp. Mat. Nauk 25:2, 265-266, 1970* ..... 197

**23 On some topological invariants of algebraic functions**
*Transact. Math. Moscow Soc. 21, 30-52, 1970. Translation of Tr. Mosk. Mat. Obsc., 27-46, 1970* ..... 199

**24 Topological invariants of algebraic functions II**
*Funct. Anal. Appl. 4, 91-98, 1970. Translation of Funkts. Anal. Prilozh. 4:2, 1-9, 1970* ..... 223

**25 Algebraic unsolvability of the problem of Lyapunov stability and the problem of topological classification of singular points of an analytic system of differential equations**
*Funct. Anal. Appl. 4, 173-180, 1970. Translation of Funkts. Anal. Prilozh. 4:3, 1-9, 1970* ..... 231

**26 On the arrangement of ovals of real plane algebraic curves, involutions of four-dimensional smooth manifolds, and the arithmetic of integral quadratic forms**
*Published as "Distribution of ovals of the real plane of algebraic curves, of involutions of four-dimensional smooth manifolds, and the arithmetic of integer-valued quadratic forms" in Funct. Anal. Appl. 5, 169-176, 1971. Translation of Funkts. Anal. Prilozh. 5:3, 1-9, 1971. Translated by Oleg Viro* ..... 239

**27 The topology of real algebraic curves (works of I.G. Petrovsky and their development)**
*Translation of Usp. Mat. Nauk 28:5, 260-262, 1973. Translated by Oleg Viro* ..... 251

**28 On matrices depending on parameters**
*Russ. Math. Surv. 22, 29-43, 1971. Translation of Usp. Mat. Nauk 26:2, 101-114, 1971* ..... 255

**29 Lectures on bifurcations in versal families**
*Russ. Math. Surv. 27, 54-123, 1972. Translation of Usp. Mat. Nauk 27:5, 119-184, 1972* ..... 271

**30 Versal families and bifurcations of differential equations (Russian)**
*Izd. Inst. Akad. Nauk Ukrain. SSR, Kiev, 42-49, 1972* ..... 341

**31 Remarks on the behaviour of a flow of a three-dimensional ideal fluid in the presence of a small perturbation of the initial vector field**
*Published as "Notes on the three-dimensional flow pattern of a perfect fluid in the presence of a small perturbation of the initial velocity field" in J. Appl. Math. Mech. 36:2, 236-242, 1972. Translation of Prikl. Mat. Mekh. 36:2, 255-262, 1972* ..... 349

**32 The asymptotic Hopf invariant and its applications**
*Selecta Math. Sov. 5:4, 327-345, 1986. Translation of Proc. All-Union School in Diff. Eq. with Infinite Number of Variables and in Dyn. Syst. with Infinite Number of Degrees of Freedom, Dilhan 1973, 229-256, 1974* ..... 357

**33 A magnetic field in a moving conducting fluid (with Ya.B. Zeldovich, A.A.Ruzmaikin, and D.D. Sokolov)**
*Translation of Usp. Mat. Nauk 36:5, 220-221, 1981. Translated by Gerald Gould* ..... 377

**34 A magnetic field in a stationary flow with stretching in a Riemannian manifold (with Ya.B. Zeldovich, A.A.Ruzmaikin, and D.D. Sokolov)**
*Published as "A magnetic field in a stationary flow with stretching in Riemannian space" in Sov. Phys. JETP 54:6, 1083-1086, 1982. Translation of Zh. Eksp. Teor. Fiz. 81:6, 2052-2056, 1981* .... 379

**35 Stationary magnetic field in a periodic flow (with Ya.B. Zeldovich, A.A.Ruzmaikin, and D.D. Sokolov)**
*Published as "Steady-state magnetic field in a periodic flow" in Sov. Phys. Dok. 27:10, 814-816, 1982. Translation of Dokl. Akad. Nauk SSSR, 266:6, 1357-1361, 1982* ..... 383

**36 Some remarks on the antidynamo theorem**
*Moscow Univ. Math. Bull. 37, 57-66, 1982. Translation of Vestn. Mosk. Univ. Ser. I, Mat. Mekh. 6, 50-57, 1982* ..... 387

**37 Evolution of a magnetic field under the action of transfer and diffusion**
*Translation of Usp. Mat. Nauk 38:2, 226-227, 1983. Translated by Gerald Gould* ..... 397

**38 The growth of a magnetic field in a three-dimensional steady incompressible flow (with E.I.Korkina)**
*Moscow Univ. Math. Bull, Ser. I, Math. Mech. 3, 50-54, 1983. Translation of Vestn. Mosk. Univ. Mat. 38:3, 43-46, 1983* ..... 399

**39 On evolution of a magnetic field under the action of drift and diffusion (Russian)**
*Some Problems in Modern Analysis, 8-21, MGU, Moscow 1984* ..... 405

**40 Exponential scattering of trajectories and its hydrodynamical applications**
*Translation of N.E. Kochin and the Development of Mechanics, 185-193, Nauka, Moscow 1984. Translated by Gerald Gould* ..... 419

**41 Kolmogorov's hydrodynamic attractors**
*Proc. Royal Soc. London A 434:1890, 19-22, 1991* ..... 429

**42 Topological methods in hydrodynamics (with B.A. Khesin)**
*Annu. Rev. Fluid Mech. 24, 145-166, 1992* ..... 433

**43 Translator's preface to J. Milnor's book "Morse Theory"**
*MIR, Moscow 1965. Translated by Gerald Gould* ..... 455

**44 Henri Poincaré: Selected Works in Three Volumes. Vol. I New Methods of Celestial Mechanics – Preface. From the editorial board. Comments**
*Nauka, Moscow 1971, 747-752 . Translated by Gerald Gould* ..... 457

**45 Comments on the paper "On a geometric theorem" by Henri Poincaré**
*H. Poincaré: Selected Works in Three Volumes. Vol. II. Klassiki Nauki, Nauka, Moscow 1972, 987-989. Translated by Gerald Gould* ..... 463

**Acknowledgements** ..... 465

**Books or book prefaces by V. Arnold written in 1965-1972, but not included in the "Collected Works"**

1. (with A. Avez): "Problèmes ergodiques de la mècanique classique" (in French), Monographies Internationales de Mathèmatiques Modernes, No. 9, Gauthier-Villars, Paris, 1967, ii+243pp.; English translation: "Ergodic problems of classical mechanics", W. A. Benjamin, Inc., New York-Amsterdam, 1968, ix+286pp.

2. "Obyknovennye differentsialnye uravneniya" (in Russian) Nauka, Moscow, 1971, 239pp.; English translation: "Ordinary differential equations", The M.I.T. Press, Cambridge, 1973, ix+280pp.; Other translations/editions of this book are French (1984), Russian ($2^{nd}$ edition, unrevised, 1975), Russian (3rd edition, revised and extended, 1984), Polish (1975, 1983), Portuguese (1985), German (1980), etc.

3. Editor's preface to a collection of papers "Singularities of differentiable mappings", Mir, Moscow, 1968.

4. Translator's preface to F. Pham's book "Introduction to the topological study of Landau's singularities", Mir, Moscow, 1970.

5. Translator's preface to J. Milnor's "Singular points of complex hypersurfaces", Mir, Moscow, 1971.