PROBLEMS for Lecture 9

9.1. Prove that for three points A, B, C on one line, where B is between A and C, one has $d(A,B) + d(B,C) = d(A,C)$.

9.2. Prove that the equality $d(A,B) + d(B,C) = d(A,C)$ implies that the points A, B, C lie on one line and B is between A and C.

9.3. Prove that the reflection in a line in the Cayley-Klein model is an involution.

9.4. Show that the notion of perpendicular lines in the Cayley–Klein model (as introduced in 9.3.2) is well defined (i.e., does not depend on the order of the two lines).

9.5. Prove that the four angles formed at the intersection point of two perpendiculars are congruent.

9.6*. Prove that the sum of angles of a triangle in the Cayley–Klein model is less than π directly from the definitions pertaining to the model.

9.7. Having defined the notion of free vector in hyperbolic geometry as suggested in 9.2.2, try to define the sum of two vectors and investigate the possibility of associating a two-dimensional vector space with hyperbolic plane geometry.

9.8. Construct a triangle in the Cayley–Klein model with angle sum less than a given positive ε.

9.9. Prove that the parallel shift T_v defined in 9.4.2 does take $(-1, 1)$ to itself and find the appropriate hyperbolic distance for which it is an isometry.