PROBLEMS for Lecture 1

1.1. List all the elements (indicating their orders) of the symmetry group (i.e., isometry group) of the equilateral triangle. List all its subgroups. How many elements are there in the group of motions (i.e., orientation-preserving isometries) of the equilateral triangle.

1.2. Answer the same questions as in Problem 1.1 for

(a) the regular n-gon (i.e., the regular polygon of n sides); consider the cases of odd and even n separately;

(b) the regular tetrahedron;

(c) the cube;

(e)* the dodecahedron;

(f)* the icosahedron;

(g) the regular pyramid with four lateral faces.

1.3. Embed the geometry of the motion group of the square into the geometry of the motion group of the cube, and the geometry of the circle into the geometry of the sphere.

1.4. For what n and m can the geometry of the regular n-gon be embedded in the geometry of the regular m-gon?

1.5. Let G be the symmetry group of the regular tetrahedron. Find all its subgroups of order 2 and describe their action geometrically.

1.6. Let G^+ be the group of motions of the cube. Indicate four subsets of the cube on which G^+ acts by all possible permutations.

1.7. Let G be the symmetry group of the dodecahedron. Indicate subsets of the dodecahedron on which G acts by all possible permutations.

1.8. Find a minimal system of generators for the symmetry group of

(a) the regular tetrahedron;

(b) the cube.

1.9. Describe fundamental domains of the symmetry group of

(a) the cube;

(b) the icosahedron;

(c) the regular tetrahedron.

1.10. Describe the Möbius band as a subset of $\mathbb{R}P^2$.

1.11. Show that the composition of two reflections of the sphere in planes passing through its center is a rotation. Determine the axis of rotation and, if the angle between the planes is given, the angle of rotation.

1.12. Given two rotations of the sphere, describe their composition.